Reputational bargaining mini course

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Introduction

Who am I?

- Jack Fanning. Associate Professor at Brown (I got tenure last year)
- PhD at NYU (2008-2014). Advisor: Ennio Stacchetti
- BA in PPE at Oxford (2003-2006)

What are we going to talk about?

- **Reputational bargaining**: key papers/ideas and my work
  - Background (pre-reputational bargaining)
  - *Abreu and Gul* (ECMA 2000)
  - *Kambe* (GEB 1999)
  - *Abreu and Pearce* (ECMA 2007)
  - My stuff and how I got there (roll back mystery of research)
    - Reputational bargaining and deadlines (ECMA 2016)
    - No compromise: uncertain costs in reputational bargaining (JET, 2018)
    - Mediation in reputational bargaining (AER, 2021)
    - Optimal dynamic mediation (JPE, 2023)
    - Outside options, reputation and the partial success of the Coase conjecture (R&R ECMA)

Background: history of bargaining theory

- *The bargaining problem:* (two) parties can work together to create surplus value. Will they do so? If so, which of many surplus divisions will they agree to?

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Axiomatic theory

- Nash solution (1950) $f(U, d) = \max_{u \in U \geq d}(u_1 - d_1)(u_2 - d_2)$
  - Undergrad thesis: Uniquely satisfies Efficiency, Symmetry, Scale Invariance, Independence to Irrelevant Alternatives
- Kalai and Smorodinsky solution (1975) $(u_1 - d_1)/(\max_{u \in U \geq d} u_1 - d_1) = (u_2 - d_2)/(\max_{u \in U \geq d} u_2 - d_2)$
  - Monotonicity instead of IIA - which axiom is more reasonable?
- Nash program (1953)
  - Justify predictions both axiomatically and non-cooperatively

Background: history of bargaining theory

Non-cooperative game theory takes over

- Rubinstein (1982). Binmore, Rubinstein & Wolinksky (1986)
  - Unique SPNE of infinite horizon alternating offer game. Converges to Nash as breakdown risk vanishes
  - Can't explain observed delay/disagreement.
  - Greatly depends on intuitively irrelevant rules: if P2 offers only in periods divisible by 3, gets at most 1/3 of surplus

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  - Seller makes all offers. Unique PBE confirms Coase conjecture: immediate agreement on $p \approx \underline{v} > 0$ if frequent offers
- Two sided private info (buyer value/seller cost). Anything goes?
  - Informed party must make offers. Can punish with beliefs off-equilibrium path: identify as highest value buyer/lowest cost seller $\Rightarrow$ low cont. payoff
  - Ausubel and Deneckere (1992): approximately no trade
  - Ausubel and Deneckere (1993): can reach Myerson & Sattertwaite (1989) efficiency bounds

Background: repeated game reputational effects

Behavioral peturbations

- Gang of four (Kreps, Wilson, Milgrom, Roberts)
  - Small possibility players are commitment types (aka commitment/obstinate/insistent/crazy types) committed to fixed strategies (private info) can drastically affect outcomes
  - e.g. chain store paradox, Kreps & Wilson (1982)

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- Fudenberg & Levine (1989, 1992)
  - One long run player vs sequence of short run players
  - Patient long run player gets Stackelberg payoff if a commitment type always plays Stackelberg action regardless of other types

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  - Patient long run player gets Stackelberg payoff if a commitment type always plays Stackelberg action regardless of other types
- No clear predictions with two equally patient long run players

Abreu & Gul (2000)

- Two player infinite horizon surplus division game
  - Players either rational or (one of many) commitment types
  - Commitment type $\alpha_i \in (0, 1)$ always demands that surplus share & won't accept less

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Key results

1. Unique equilibrium as offers become frequent regardless of offer protocol details and despite 2-sided private info
2. War-of-attrition with delay: rational types imitate commitment types before eventually conceding
3. If rich set of commitment types, then payoffs converge to alternating offers game payoffs as commitment vanishes

Single type, continuous time, war-of-attrition

- Player i is commitment type with prob $z_i \in (0,1)$
- Single commitment type demands surplus share $\alpha_i \in (0,1)$ where $\alpha_1 + \alpha_2 > 1$
- Rational player can *concede* to opponent demand at any $t \in [0,\infty]$
  - If i alone concedes to j at t then game ends with shares $(1-\alpha_j, \alpha_j)$
  - If both players concede at same time, each demand selected with prob $1/2$
  - Rational i payoff from share x at time t is: $e^{-r_i t}x$, where $r_i > 0$
- Implicit description of strategy:

$$F_i(t) = Pr[i, \text{ whether rational or committed, conceded at } s \leq t]$$

- Reputation:

$$\bar{z}_i(t) = \frac{z_i}{1 - F_i(t)}$$

Single type, continuous time, war-of-attrition

Proposition 1

Unique Nash equilibrium characterized by three properties:

i) *At most one player concedes with prob* > 0 *at time 0*

ii) *Both players' reputation reach 1 at same time* $T^* \in (0, \infty)$

iii) *Rational player* i *concedes at constant rate* λ_i *on* $(0, T^*]$ *to make a rational opponent* j *indifferent to conceding*

$$\frac{f_i(t)}{1 - F_i(t)} = \lambda_i = \frac{r_j(1 - \alpha_i)}{\alpha_j + \alpha_i - 1}$$

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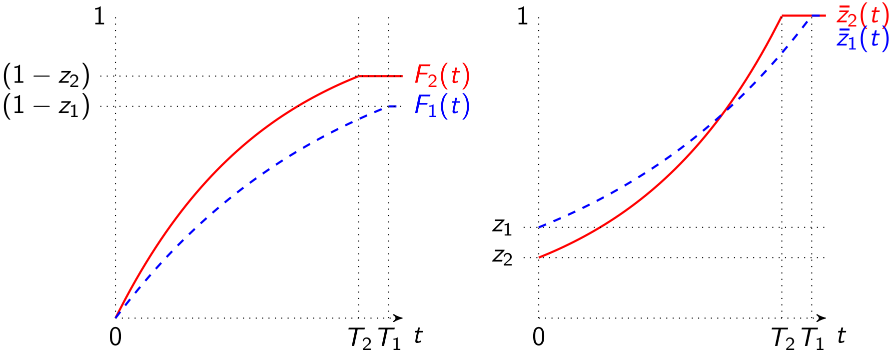
Why?

(i) If i concedes w prob> 0 at time t then j gets higher payoff conceding just after t than from conceding on $[t - \varepsilon, t]$

(ii) If rational player i ever knows opponent j committed then immediately concedes

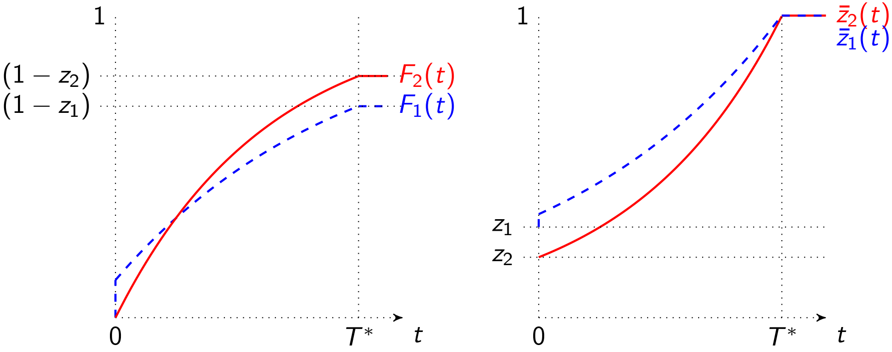
(iii) Concession continuous at $t > 0$ by (i). To motivate interval w/o concession by i we'd need discontinuous concession by j at end of interval. So indifferent to conceding on $(0, T^*)$

Reputational race: if no time 0 concession



- $T_i = -ln(z_i)/\lambda_i$ is time to reach reputation 1 if no concession at 0

Reputational race: adjust time 0 concession



- $T_i = -ln(z_i)/\lambda_i$ is time to reach reputation 1 if no concession at 0
- Adjust $t = 0$ concession so both reputations=1 at $T^* = \min\{T_1, T_2\}$

$$1 - F_i(0) = z_i e^{\lambda_i T^*} = \min\{1, z_i z_j^{-\lambda_i/\lambda_j}\}$$

- Payoffs: $U_i = F_j(0)\alpha_i + (1 - F_j(0))(1 - \alpha_j)$
- Here: $r_i = 1$, $\alpha_1 = 2/3$, $\alpha_2 = 1/2$, $z_1 = 0.3$, $z_2 = 0.2$ so $\lambda_2 = 3 > \lambda_1 = 2$

What happens as commitment vanishes?

Proposition 2

Consider any sequence of bargaining games (z_i^n, r_i, α_i) *where players' commitment vanishes at the same rate.* If* $\lambda_j > \lambda_i$ *then* i *immediately concedes with probability approaching 1.*

* $z_i^n \to 0$ with $z_1^n / z_2^n \in [1/L, L]$ for some $L \geq 1$

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Why?

$$1 - F_i(0) = z_i e^{\lambda_i T^*} = \min\{1, z_i z_j^{-\lambda_i/\lambda_j}\}$$

- As $z_i^n \to 0$, takes a long time to reach reputation 1: $T_i^n \to \infty$
  - Constant concession rate$\Rightarrow$don't concede with probability 1 in finite time $F_i(t) = 1 - e^{-\lambda_i t} < 1$
- Reputation growth rate=concession rate $\frac{d\bar{z}_i(t)/dt}{\bar{z}_i(t)} = \lambda_i$
- If j's reputation grows exponentially faster than i's over long interval, then i must immediately concede with high prob so both reach reputation 1 at same time T^*
- *Intuitive:* $\lambda_j = r_i(1-\alpha_j)/(\alpha_i + \alpha_j - 1) > \lambda_i$ iff i has higher cost of delay $r_i(1-\alpha_j) > r_j(1-\alpha_i)$
- N.B. If $\alpha_j \leq r_i/(r_i + r_j)$ then $r_i(1-\alpha_j) > r_j(1-\alpha_i)$

Generalized model: demand choice

- Finite set of commitment types $C_i \subset (0,1)$
  - Conditional on commitment, player i is of type α_i with prob $\pi_i(\alpha_i)$
- Player 1 announces demand $\alpha_1 \in C_1$

$$\mu_1(\alpha_1) = Pr[\text{ rational player 1 demands } \alpha_1]$$

- Player 2 either accepts, or counterdemands $\alpha_2 \in C_2$ (causing WOA)

$$\mu_2^{\alpha_1}(\alpha_2) = Pr[\text{ rational player 2 demands } \alpha_2|\alpha_1]$$

- Updated reputations:

$$\bar{z}_1(\alpha_1) = \frac{z_1\pi_1(\alpha_1)}{z_1\pi_1(\alpha_1) + (1-z_1)\mu_1(\alpha_1)}, \quad \bar{z}_2^{\alpha_1}(\alpha_2) = \frac{z_2\pi_2(\alpha_2)}{z_2\pi_2(\alpha_2) + (1-z_2)\mu_2^{\alpha_1}(\alpha_2)}$$

Generalized model: demand choice

Proposition 3

There is an essentially unique equilibrium (all eq. have same distribution of outcomes)

- Due to form of strategic substitutability: as i demands α_i more often, she receives lower cont payoff
    - Player i WOA payoff is increasing in $\bar{z}_i$ and decreasing in $\bar{z}_j$

Generalized model: demand choice

Proposition 4

Consider any sequence of bargaining games (z_i^n, r_i, C_i, π_i) *where players' commitment vanishes at the same rate.* If* $\alpha_i' \leq r_j/(r_i + r_j)$ *for* $\alpha_i' \in C_i$ *then* $\liminf_n U_i^n \geq \alpha_i'$.

- If rich set of demands then almost immediate agreement on frequent alternating offer division
  - Alternating offers game with period length Δ has $\delta_i = e^{-r_i \Delta}$:

$$U_1 = 1 - U_2 = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} = \frac{1 - e^{-r_2 \Delta}}{1 - e^{-(r_2 + r_1)\Delta}} \to \frac{r_2}{r_1 + r_2}$$

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Intuition:

- Alternating offers equalizes players' off-path cost of delay (if not...)
- If $r_j/(r_i + r_j) \geq \alpha_i' > 1 - \alpha_j$ in reputational model then j has higher cost of delay, $r_j(1-\alpha_i') > r_i(1-\alpha_j)$ so immediately concedes as commitment vanishes

Generalized model: demand choice

Proposition 4

Consider any sequence of bargaining games $(z_i^n, r_i, \alpha_i, C_i, \pi_i)$ where players' commitment vanishes at the same rate. If $\alpha_i' \leq r_i/(r_i + r_j)$ for $\alpha_i' \in C_i$ then $\liminf_n U_i^n \geq \alpha_i'$.*

* $z_i^n \to 0$ with $z_1^n / z_2^n \in [1/L, L]$ for some $L \geq 1$

Proof (for player 2)

- If P1 demands $\alpha_1 > 1 - \alpha_2'$ with $\lim_n \mu_1^n(\alpha_1) > 0$ then even if P2 always demands α_2' must have $\bar{z}_1^n \to 0$ and $z_1^n / \bar{z}_2^n \to 1/\lim_n \mu_1^n(\alpha_1)$

$$\bar{z}_1(\alpha_1) = \frac{z_1 \pi_1(\alpha_1)}{z_1 \pi_1(\alpha_1) + (1 - z_1)\mu_1(\alpha_1)}, \quad \bar{z}_2^{\alpha_1}(\alpha_2) = \frac{z_2 \pi_2(\alpha_2)}{z_2 \pi_2(\alpha_2) + (1 - z_2)\mu_2^{\alpha_1}(\alpha_2)}$$

- Since $\alpha_2' \leq r_2/(r_1 + r_2)$ we have $\lambda_2 > \lambda_1$ in WOA so player 1 immediately concedes in limit by Proposition 2

Generalized model: discrete time

- Discrete time games G^n with fixed fundamentals (z_i, r_i, C_i, π_i)
- Periods $m \in \mathbb{N}$ correspond to real times $t^n(m) \in [0, \infty)$
- In each real time interval $[t, t + \Delta^n]$ each player can make an offer
  - Sequential or simultaneous offers, j can make lots more than i
- *Distribution of sequential equilibrium outcomes:* θ^n in G^n, θ in cont time game

Generalized model: discrete time

Proposition 5

For any sequence of discrete time games G^n and distributions of equilibrium outcomes θ^n with $\Delta^n \to 0$, we have $\theta^n \to_w \theta$.

- Predictions don't depend on details of bargaining protocol!
- Unique eq. limit despite 2-sided private info!
  - Commitment types immune to belief punishments: force behavior onto eq. path
- Subsequent literature often jumps straight to cont. time game
  - Not always well-motivated...

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Why?

- **Reputational Coase conjecture:** if $\Delta^n \approx 0$ and player i reveals rationality by t but j hasn't, then i concedes almost immediately.
  - More limited result in Myerson (1991)
  - For any $\varepsilon > 0$, there exists $\bar{\Delta} > 0$ such that if $\Delta^n \leq \bar{\Delta}$ then cont. payoffs $U_i \leq 1 - \alpha_j + \varepsilon$ and $U_j \geq \alpha_j - \varepsilon$

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- Player i must concede before some $T^n < \infty$ (if j not revealed)
  - If don't concede at $s \geq t$, for any $p \in (0, 1 - \alpha_j)$ exists $K > 0$ s.t j must reveal w prob$\geq p$ on $[s, s+K]$: $(1 - \alpha_j) = p + (1-p)e^{-r_i K}$
  - Bayesian updating: $\bar{z}_j(t + LK) \geq z_j \pi_j(\alpha_j)/(1-p)^L \to \infty$?

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  - Bayesian updating: $\bar{z}_j(t + LK) \geq z_j \pi_j(\alpha_j)/(1 - p)^L \to \infty$?
- If $\lim_n T^n = T > t$ then for small $\varepsilon > 0$ and $\beta \in (0, 1)$ and large n, player j must reveal with prob $q > 0$ on $[T - \varepsilon, T - \beta\varepsilon]$
  - Since rational j can guarantee $e^{-r_j(T-s)}\alpha_j$ at s by waiting for T need

$$1 - \alpha_j \leq q(1 - e^{-r_j \epsilon}\alpha_j) + (1 - q)e^{-r_i \varepsilon(1-\beta)}(1 - e^{-r_j \beta \epsilon}\alpha_j)$$

- Repeating argument on $[T - \beta^L \varepsilon, T - \beta^{L+1}\varepsilon]$, Bayesian updating gives $\bar{z}_j(T - \beta^L \varepsilon) \geq z_j \pi_j(\alpha_j)/(1 - q)^L \to \infty$?

Kambe (1999)

- Adaption of Abreu & Gul [AG], but published before
- *All players initially rational:* simultaneously announce any demand $\alpha_i \in [0,1]$, then become committed w prob $z_i \in (0,1)$
  - No punishment with beliefs b/c no private info when announce
- If $\alpha_1 + \alpha_2 > 1$ then reputational WOA

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Key results

i) In any equilibrium where players don't mix over demands: immediate agreement with

$$U_i = \alpha_i = \frac{ln(z_j)r_j}{ln(z_j)r_j + ln(z_i)r_i}$$

ii) If commitment vanishes at the same rate for both players then in all eq payoffs $U_i^n \to r_j/(r_i + r_j)$

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Key results

i) In any equilibrium where players don't mix over demands: immediate agreement with

$$U_i = \alpha_i^* = 1 - \alpha_j^* = \frac{ln(z_j)r_j}{ln(z_j)r_j + ln(z_i)r_i} \qquad (1)$$

ii) If commitment vanishes at the same rate for both players then in all eq payoffs $U_i^n \to r_j/(r_i + r_j)$

Why?

(i) Rearranging (1) gives: $z_i z_j^{-r_j(1-\alpha_i^*)/(r_i(1-\alpha_j^*))} = 1$ so if $\alpha_i = \alpha_i^* > 1 - \alpha_j$ WOA satisfies

$$1 - F_i(0) = z_i e^{\lambda_i T^*} = \min\{z_i z_j^{-r_j(1-\alpha_i^*)/(r_i(1-\alpha_j))}, 1\} = 1$$

$$U_j = (1 - \alpha_i^*)[1 - z_i z_j e^{-r_j T^*}] < 1 - \alpha_i^* = \alpha_j^*$$

For arbitrary $\alpha_i + \alpha_j > 1$, have $F_i(0) = 0$ for some i so $U_j < (1 - \alpha_i)$. Hence $\alpha_j' = 1 - \alpha_i$ is profitable deviation

(ii) Demanding $\alpha_i^{*,n}$ gives lower bound on profits $U_i^n \to r_j/(r_i + r_j)$

Abreu & Pearce (2007)

- Two player repeated game with contracting (equal discounting)
- Repeatedly play stage game while offering enforceable contract for long term behavior
- Commitment types adopt *time-varying, history contingent* strategy
  - e.g. change pre-contract game behavior and contract offer depending on opponent play
- Without commitment types - folk theorem

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Key result

- If player has Nash-bargaining-with-threats (NBWT) type then can guarantee NBWT payoff as commitment vanishes
- Nash (1953) defines NBWT in simple game
  1. Players first simultaneously announce strategies for stage game ("threats")
  2. Then Nash bargaining over stage game contract with payoffs from (1) if disagree

Abreu & Pearce (2007)

Key result: If player has Nash-bargaining-with-threats (NBWT) type then can guarantee NBWT payoff as commitment vanishes

- Assume *stationary commitment types*: pre-contract stage game behavior⇒flow disagreement payoffs d
- Offer feasible flow payoffs $u^i = (u^i_1, u^i_2)$ then WOA. Concession rate:

$$\frac{f_i(t)}{1 - F_i(t)} = \lambda_i = \frac{r(u^i_j - d_j)}{u^j_j - u^i_j}$$

- If i's type offers Nash division $u^i = \arg\max_{u \in U}(u_1 - d_1)(u_2 - d_2)$ then guarantee u^i_i as $z^n_i \to 0$, because

$$\begin{aligned}(\lambda_i - \lambda_j)\frac{(u^i_i - u^j_i)(u^j_j - u^i_j)}{r} &= (u^i_j - d_j)(u^i_i - u^j_i) - (u^j_i - d_i)(u^j_j - u^i_j) \\ &= (u^i_j - d_j)(u^i_i - d_i) - (u^j_i - d_i)(u^j_j - d_j) > 0\end{aligned}$$

- *Intuition:* Efficient, Symmetric, Scale Invariance, IIA

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$$\begin{aligned}(\lambda_i - \lambda_j)\frac{(u^i_i - u^j_i)(u^j_j - u^i_j)}{r} =& (u^i_j - d_j)(u^i_i - u^j_i) - (u^j_i - d_i)(u^j_j - u^i_j) \\ =& (u^i_j - d_j)(u^i_i - d_i) - (u^j_i - d_i)(u^j_j - d_j) > 0\end{aligned}$$

- *Intuition:* Efficient, Symmetric, Scale Invariance, IIA
- Given this, rational players would choose fixed NBWT threats type

Abreu & Pearce (2007)

Key result: If player has Nash-bargaining-with-threats (NBWT) type then can guarantee NBWT payoff as commitment vanishes

Stationary types are rich enough

- Non-stationary threat/demand? Still immediately concede vs NBWT opponent as $z_i^n \to 0$
  - Increasing j demand increases i's SR delay cost, but j has higher LR delay cost and *only LR costs matter*
  - If no concession at $t = 0$ then for any $T \in (0, \infty)$ we have $1 - \lim_n F_i^n(T) \geq \varepsilon_T > 0$, despite non-constant concession rate
  - So $\bar{z}_i^n(T) = \bar{z}_i^n/(1 - F_i^n(T)) \to 0$ and $\bar{z}_j^n(T)/\bar{z}_i^n(T) \in [1/L, L]$ for some $L > 1$

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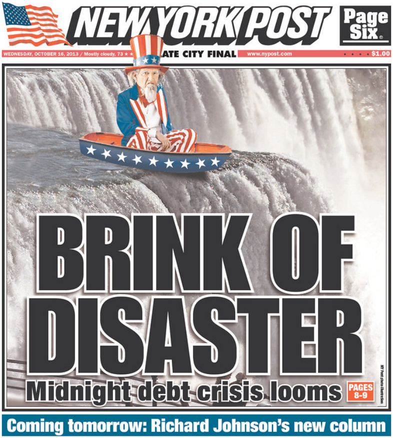
Stationary types are rich enough

- Non-stationary threat/demand? Still immediately concede vs NBWT opponent as $z_i^n \to 0$
  - Increasing j demand increases i's SR delay cost, but j has higher LR delay cost and *only LR costs matter*
  - If no concession at $t = 0$ then for any $T \in (0, \infty)$ we have $1 - \lim_n F_i^n(T) \geq \varepsilon_T > 0$, despite non-constant concession rate
  - So $\bar{z}_i^n(T) = \bar{z}_i^n/(1 - F_i^n(T)) \to 0$ and $\bar{z}_j^n(T)/\bar{z}_i^n(T) \in [1/L, L]$ for some $L > 1$
- Result relies on transparent commitment types: truthfully announce strategy at $t = 0$
  - Wolitzky (2011) highlights problems with non-transparent types (initially pretend to be NBWT)

My papers: Reputational bargaining and deadlines (2016)

Bargaining in the face of a **deadline** when parties might be **obstinate**



- 2011: "Crazy" House Republicans refuse to increase debt ceiling until Obama accepts spending cuts in 11^{th} hour deal
- 2013 reprise of crisis

My papers: Reputational bargainng & deadlines (2016)

- Uncertain deadline on $[0, T]$ with $T < \infty$
  - $G(t) = Pr[$ deadline arrives at $s \leq t]$
  - Continuous positive delay cost: $r_i + g(t) > 0$
  - Small uncertainty? $G(T - \varepsilon) = 0$
- Divide "dollar"
  - Utility of money $u_i(m)$ with $u_i' > 0$, $u_i'' \leq 0$.
  - Get $u_i(d_i) \geq 0$ if deadline hits
- Stationary commitment types demand share $\alpha_i \in (d_i, 1 - d_i)$
- Non-stationary commitment type demand function
  - Can be smooth, discontinuous, history contingent

My papers: Reputational bargainng & deadlines (2016)

Result 1 (Deadline effects)

If patient players, small deadline uncertainty ($r_i^n \to 0$, $G^n \to \delta_T$) and only stationary commitment types then **deadline effects***: U-shaped agreement+rational disagreement*

$$\lambda_i(t) = \frac{r_j u_j(1-\alpha_i) + (u_j(1-\alpha_i) - u_j(d_j))g(t)/(1-G(t))}{u_j(\alpha_j) - u_j(1-\alpha_i)}$$

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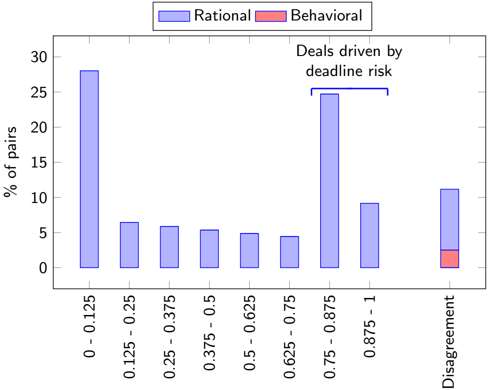

Predicted agrement times and disagreement*

* $G(t) = \mathbb{1}_{[t>0.75]}(t-0.75)^2/0.0625, r_i = 0.25, z_1 = 0.25, z_2 = 0.1, \alpha_1 = 0.75, \alpha_2 = 0.5$

My papers: Reputational bargainng & deadlines (2016)

Result 2 (Optimal stationary demands)

If only stationary commitment types whose prob vanishes, rational player can obtain **Nash bargaining payoff** *by imitating Nash demand type (even if* $r_i >> r_j$*)*

My papers: Reputational bargainng & deadlines (2016)

Result 2 (Optimal stationary demands)

If only stationary commitment types whose prob vanishes, rational player can obtain **Nash bargaining payoff** *by imitating Nash demand type (even if $r_i >> r_j$)*

Why?

- Like Abreu&Pearce: only LR delay costs matter (close to T) as $z_i^n \to 0$
- Impatience irrelevant for $t \approx T$ when $g(t)/(1 - G(t)) \approx \infty$
  - $\lambda_i(t) - \lambda_j(t) \approx \infty$ iff higher Nash product for i proposal

$$\lambda_i(t) = \frac{r_j u_j(1 - \alpha_i) + (u_j(1 - \alpha_i) - u_j(d_j))g(t)/(1 - G(t))}{u_j(\alpha_j) - u_j(1 - \alpha_i)}$$

My papers: Reputational bargainng & deadlines (2016)

Result 3 (Optimal non-stationary demands)

If non-stationary commitment types whose prob vanishes, then rational player can obtain **generalized Rubinstein** *payoff by imitating generalized Rubinstein demand type*

- $\alpha_i^R(t)$ =demand in complete info alternating offers game for deadline environment

$$2\alpha_i'^R(t) = \frac{r_i u_i(\alpha_i^R(t)) + \frac{g(t)}{1-G(t)}(u_i(\alpha_i^R(t)) - u_i(d_i))}{u_i'(\alpha_i^R(t))} - \frac{r_j u_j(1-\alpha_i^R(t)) + \frac{g(t)}{1-G(t)}(u_j(1-\alpha_i^R(t)) - u_j(d_j))}{u_j'(1-\alpha_i^R(t))}$$

- Convex combination of infinite horizon Rubinstein and Nash demands. Approaches Nash as $t \to T$

My papers: Reputational bargainng & deadlines (2016)

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- Convex combination of infinite horizon Rubinstein and Nash demands. Approaches Nash as $t \to T$

Why?

- AG: Alternating offers equalizes players' off-path cost of delay...
- Uniquely ensures $\lambda_1(t)/\lambda_2(t) = 1$ for $\alpha_1 \to 1 - \alpha_2 \in [0,1]^{[0,T]}$
- As in Abreu&Pearce in SR can have $\lambda_2(t) > \lambda_1(t)$ despite $\alpha_1 = \alpha_1^R$ but then $\lambda_2(t) << \lambda_1(t)$ in LR

$$\lambda_i(t) = \frac{r_j u_j(1-\alpha_i) + (u_j(1-\alpha_i) - u_j(d_j))g(t)/(1-G(t)) + \alpha_i'(t)u_j'(1-\alpha_i(t))}{u_j(\alpha_j) - u_j(1-\alpha_i)}$$

How project started...

Econometrica, Vol. 68, No. 1 (January, 2000), 85–117

BARGAINING AND REPUTATION

By Dilip Abreu and Faruk Gul[1]

The paper develops a reputation based theory of bargaining. The idea is to investigate and highlight the influence of bargaining "postures" on bargaining outcomes. A complete information bargaining model à la Rubinstein is amended to accommodate "irrational types" who are obstinate, and indeed for tractability assumed to be completely inflexible in their offers and demands. A strong "independence of procedures" result is derived: after initial postures have been adopted, the bargaining outcome is independent of the fine details of the bargaining protocol so long as both players have the opportunity to make offers frequently. The latter analysis yields a unique continuous-time limit with a war of attrition structure. In the continuous-time game, equilibrium is unique, and entails delay, consequently inefficiency. The equilibrium outcome reflects the combined influence of the rates of time preference of the players and the ex ante probabilities of different irrational types. As the probability of irrationality goes to zero, delay and inefficiency disappear; furthermore, if there is a rich set of types for both agents, the limit equilibrium payoffs are inversely proportional to their rates of time preference.

Keywords: War of attrition, delay, incomplete information, independence from procedures, obstinate types.

1. INTRODUCTION

This paper addresses the following question. Two agents seek to divide some surplus: to what division will they agree? Our approach is to emphasize the role of *reputation* in the determination of this division.

Noncooperative bargaining theory in its current form has been deeply influenced by the celebrated paper of Rubinstein (1982), which has provided the basic framework for an enormous and still growing literature. His paper provides a natural reference point for our own work. The only parameters in Rubinstein's *complete* information model are the players' costs of waiting (due to impatience, exogenous termination, etc.) for their turn to make an offer. These parameters determine a unique equilibrium.

Our theory replaces the *impatience* between offers of Rubinstein by uncertainty about the *strategic posture* of one's opponent. Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), we have "irrational" types where each type is identified by a fixed offer and acceptance rule. While players are still impatient, the driving force of the theory is the peripheral uncertainty about the inflexible demand (or rule of thumb, bargaining convention, et cetera) with which one's opponent may be endowed, or more significantly, to which one's opponent may pretend.

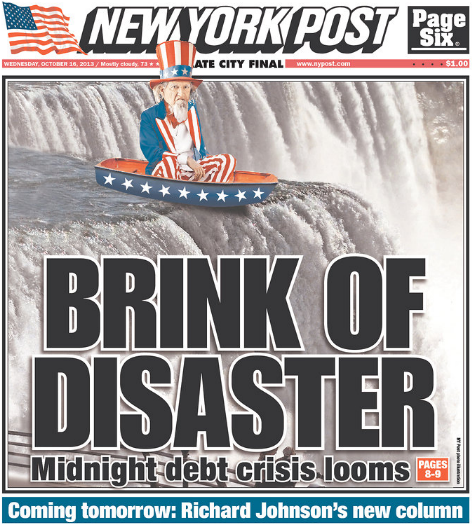
[1]Abreu thanks the Russell Sage Foundation and Gul thanks the Alfred P. Sloan Foundation for their generous support. Both authors gratefully acknowledge financial support from the National Science Foundation.

- Year 2 of PhD: New Research in Economic Theory (NRET) seminar
- Chose from list (compiled by Ennio Stacchetti) of candidate papers to present

What happened next...

- Not much reputational bargaining...
- Next NRET presentation: Sannikov (2008)
- 2nd year research paper: principal agent model (counting contracts)
- 3rd year paper attempts:
    - competition for political ideas,
    - bad bosses/sabotage,
    - eventually: experimental economics *Finding cooperators: sorting through repeated interaction* with Sevgi Yuksel & Mark Bernard, JEBO (2018)

What happened next...



NEW YORK POST

Page Six

WEDNESDAY, OCTOBER 16, 2013 / Mostly cloudy, 73

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BRINK OF DISASTER

Midnight debt crisis looms

PAGES 8-9

Coming tomorrow: Richard Johnson's new column

- Fourth year: a return to reputational bargaining motivated by debt ceiling crisis (2011)

First ideas

- To me: looked v like AG, except what difference did deadline make?

A first model:

- Deadline arrival distribution G on $[0, T]$, $r_i = 0$, $d_i = 0$, $u_i(x) = x$, stationary commitment types

$$\frac{f_i(t)}{1 - F_i(t)} = \frac{g(t)(1-\alpha_i)}{(1-G(t))(\alpha_1 + \alpha_2 - 1)}, \qquad \frac{1 - F_i(t)}{1 - F_i(0)} = (1 - G(t))^{(1-\alpha_i)/(\alpha_1+\alpha_2-1)}$$

- Agreements just tracked G. Agents could obtain $1/2$ when $z_i^n \to 0$.
- Did not seem super interesting/different from AG...

Moving on, too quickly

- *Next idea:* Negotiate over multiple issues with different deadlines

Moving on, too quickly

- *Next idea:* Negotiate over multiple issues with different deadlines
- Fortunately, David Pearce advised me to *slow down*, and further explore deadline model
  - Effect of impatience vs deadline, non-stationary demands
  - Developed papers limiting results as $z_i^n \to 0$
- 5^{th} year again moved onto new paper: seller vs private value buyer; fairness preferences
  - Delay due to interdependent values, but too like Denekere and Liang (2006)
  - Ignored for long time after PhD, before published JOEP (2022)
- Back to deadline model (just before JM): Fortunately Debraj Ray urged me to refocus on initial motivation (debt ceiling) with formal deadline effects result
- Fortunately, *another* debt ceiling crisis during JM (Nov 2013)
- Referee: general $u_i(x)$, $d_i > 0$ (Nash vs 50/50)

No compromise: uncertainty in reputational bargaining (JET, 2018)

Motivation:

- Bargaining delays are common⇒impose substantial costs
- Deadlocks often broken by arrival of news about those costs

Some examples

- 2013 government shutdown: release of polls blaming Republicans led to retreat on demands to dismantle Obamacare
- 2012 presidential election and Fiscal Cliff: Republicans back down given prospect of press blame if resist Obama's "mandate" for tax rises
  - Explains *Mayhew (2003)*: significantly more important legislation 2 years after presidential election vs. 2 years before
- *Aim of paper:* investigate whether interaction of reputation and uncertainty/arrival of news can explain such delay
- *Origin of paper:* outgrowth of model with 2 deadlines for 2 issues, with uncertain values for 2nd issue (developed 1st year Brown)

Headline result

- Significant delay even if vanishingly small probability of commitment
  - Rational agents demands polarize
  - Deadlock broken by news about costs
- Surprising!
  - More delay than if all are obstinate (demands sometimes compatible)
  - Contrast with no delay in complete info limit in AG.

The model: simplest version

- Two agents, divide dollar, infinite horizon
- Before some *revelation time*, R, agent i faces flow cost of delaying agreement, $x_0^i > 0$
  - Utility if i gets α^i at $t \leq R$ is $\alpha^i - x_0^i t$
  - Time R divided into R_{-1}, R_0, R_{+1}: no delay costs between

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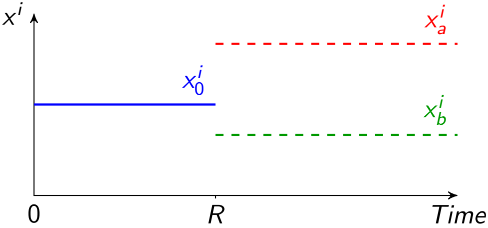
  - Time R divided into R_{-1}, R_0, R_{+1}: no delay costs between
- At time R_0, the state of the world $\omega \in \Omega$ is revealed, determining cost of delay $x_\omega^i > 0$, from R_{+1} onwards
  - Utility from α^i at $t \geq R$ in state ω is $\alpha^i - x_0^i R - x_\omega^i (t - R)$
  - Ω is finite, probability measure p

Stationary commitment types: $\alpha^i \in (0, 1)$

A helpful picture

- An example with two states: $\Omega = \{a, b\}$, $p(a) = 1 - p(b) \in (0, 1)$



Work backwards: equilibrium **without** uncertainty

- AG: *unique equilibrium* with known costs and demands
  - Equivalent to my continuation game at R_{+1} in state ω

Continuation payoffs (at revelation time):

$$\text{At } R_{+1}: \quad V_\omega^i = Pr[j \text{ concedes at } R_{+1}]\alpha^i + Pr[j \text{ doesn't concede at } R_{+1}](1-\alpha^j)$$

$$\text{At } R_0: \quad V^i = \mathbb{E}_p V_\omega^i$$

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$$\text{At } R_0: \quad V^i = \mathbb{E}_p V^i_\omega$$

Use to demonstrate *unique equilibrium* (before revelation time):

1. At most one agent concedes at time zero
2. Agents indifferent to concession on $(0, \hat{T})$. Concession rate:

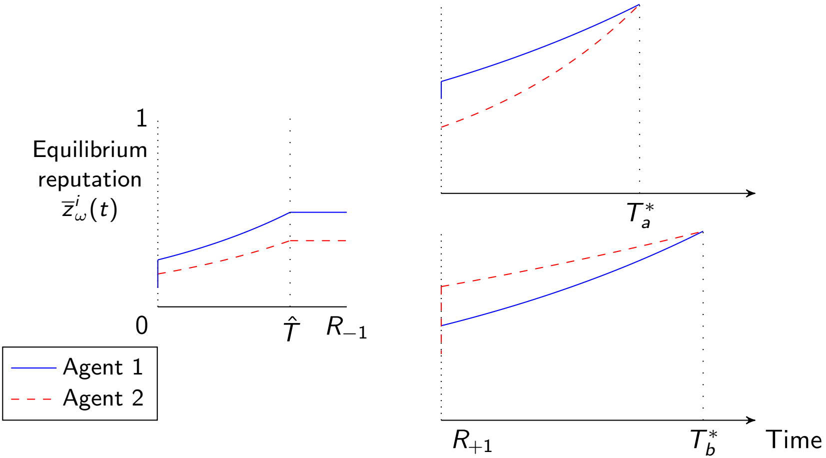
$$\frac{f^j_0(t)}{1-F^j_0(t)} = \lambda^j_0 := \frac{x^i_0}{\alpha^j + \alpha^i - 1}$$

3. At $\hat{T}$ either:
   - i Both agents reach a probability 1 reputation
   - ii Both agents wait until revelation time R

$$U^i|\alpha = F^j_0(0)\alpha^i + (1-F^j_0(0))max\left\{1-\alpha^j, V^i - x^i_0 R\right\}$$

In pictures: “typical” equilibrium with uncertainty

- Agent 1 concedes at 0 and at R_{+1} in state a
- Agent 2 concedes at R_{+1} in state b



Complete information limit: after the revelation time

What happens if initial reputations are small?

- AG: immediate agreement without uncertainty

Complete information limit: after the revelation time

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- AG: immediate agreement without uncertainty
- Suppose reputations at R_{-1} vanish at same rate:
  - $\bar{z}_0^i(R_{-1}) \to 0$ with $L > \frac{\bar{z}_0^1(R_{-1})}{\bar{z}_0^2(R_{-1})} > \frac{1}{L}$
- Agent j concedes at R_{+1} with probability≈1 in state ω iff $x_\omega^j > x_\omega^i$

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- Limit continuation payoff:

$$\tilde{V}^i = Pr[x_\omega^j > x_\omega^i]\alpha^i + Pr[x_\omega^j < x_\omega^i](1 - \alpha^j)$$

Complete information limit: before the revelation time

- Assume only a single demand/type for each agent
  - Define *limit waiting time*, $\tilde{T}_W^i$
  - What is longest interval $[\tilde{T}_W^i, R]$ on which agent i would wait to receive $\tilde{V}^i$ rather than $(1-\alpha^j)$ immediately?

$$\tilde{V}^i - (R - \tilde{T}_W^i)x_0^i = (1-\alpha^j)$$

$$\tilde{T}_W^i = R - \frac{Pr[x_\omega^j > x_\omega^i]}{x_0^i}(\alpha^i + \alpha^j - 1)$$

Complete information limit: before the revelation time

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$$\tilde{V}^i - (R - \tilde{T}^i_W)x^i_0 = (1-\alpha^j)$$

$$\tilde{T}^i_W = R - \frac{Pr[x^j_\omega > x^i_\omega]}{x^i_0}(\alpha^i + \alpha^j - 1)$$

- Uniquely characterizes limit outcomes:
  1. j concedes at 0 with probability≈1 if $\tilde{T}^j_W > max\{\tilde{T}^i_W, 0\}$
  2. Delay until R with probability≈1 if $max\{\tilde{T}^j_W, \tilde{T}^i_W\} < 0$

Demand choice

- If demands are polarized ($\alpha^i >> 1 - \alpha^j$) delay until R is possible
- Without polarized demands, delay is impossible.
  - Gain of α^i compared to $1 - \alpha^j$, not worth large cost

$$\tilde{T}^i_W = R - \frac{Pr[x^j_\omega > x^i_\omega]}{x^i_0}(\alpha^i + \alpha^j - 1)$$

- Must consider demand choice
  - Allow many different commitment types/demands

Definition 1

A type space is ε-rich if for each $a \in [0, 1]$ there is some $\alpha^i \in C^i$ such that $|\alpha^i - a| < \varepsilon$.

What happens?

- Intuition: incentive to avoid delay by moderating demands?

What happens?

- Intuition: incentive to avoid delay by moderating demands?
- In fact: Delay in complete information limit. Rational agents *choose* to polarize!

Example with delay and polarization

- Suppose $Pr[x^i_\omega > x^j_\omega] = \frac{1}{2}$, while $x^1_0 > x^2_0$ and $R < \frac{1}{4x^1_0 + 2x^2_0}$
- Prior probability of commitment types vanishes at same rate for both agents. Rich type space.
- *Prediction:*
  - Demands: $\alpha^i = 1$
  - Limit continuation payoffs: $\tilde{V}^i = \frac{1}{2}$
  - Equilibrium payoffs: $U^i = \frac{1}{2} - x^i_0 R$

Example contd.

- Delay is inefficient $\frac{1}{2} - x_0^i R < \frac{1}{2}$
- Both agents prefer compromise outcome $(\frac{1}{2}, \frac{1}{2})$. Why not develop reputation $\alpha^i = \frac{1}{2}$?

Example contd.

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  - Increases opponent's option value $\tilde{V}^j$ of waiting to R
  - Can't secure agreement on compromiser's terms
- Consider deviation by 2
  - Suppose $\alpha^1 = 1$, but $\alpha^2 \approx \frac{1}{2}$ (with positive limit probability)
  - Increases 1's immediate value to conceding (from 0 to $\frac{1}{2}$)
  - But *also* increases 1's option value of waiting (from $\frac{1}{2} - x_0^1 R$ to $\frac{3}{4} - x_0^1 R$)
  - Goalposts shift. 2 won't accept: $\frac{3}{4} - x_0^1 R > \frac{1}{2}$
- *Underlying problem:* agents can't adjust demand to fit environment without sacrificing reputation for obstinacy

General characterization

Main result

- Characterize complete information limit for all parameters
  - For agents a and b let $p = Pr[x_\omega^b > x_\omega^a]$ (wlog $px_0^b > (1-p)x_0^a$)
  - Always delay and polarization when $R < \frac{p(1-p)}{x_0^a + x_0^b(1+p)}$
  - For slightly larger R, delay only if b announces demand first
  - For large R always agreement immediately
  - Inefficiency may amount to half the surplus!

Other results

- Model doesn't require flow costs, agents can discount payoffs exponentially instead
  - Examples with delay and similar (if not complete) polarization
- Delay predictions also extend to Kambe (1999) model

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  - Stackelberg leader logic
  - Choose demand which threatens delay. Incumbent on agent 2 to eliminate inefficiency with generous counteroffer
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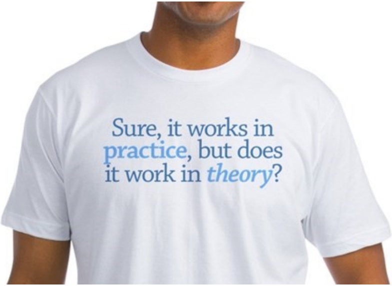
  - Choose demand which threatens delay. Incumbent on agent 2 to eliminate inefficiency with generous counteroffer
- Or a second mover advantage
  - Can optimize counterdemand using more information
- More general commitment types can be committed to time/state varying demand strategies
  - Outcomes again converge to alternating offers solution

Mediation in reputational bargaining (2021)

What is mediation?

- A third party (mediator) helps conflicting parties reach a *voluntary agreement*. Distinct from arbitration which can impose agreement
- *Where used?*
  - International conflicts, industrial relations, an alternative to court
    - Dixon (1996): Mediation efforts in 13% of "phases" of international conflicts 1947-1982
    - Stripanowich & Lamare (2013): 42%+ of Fortune 1000 companies always/often use mediation (vs 17%- arbitration)
- *Why used?*
  - Dixon (1996): Mediated disputes 47% less likely to escalate, 24% more likely to peacefully resolve (vs no conflict management)
  - Emery, Matthews & Wyer (1991), randomized controlled trial: mediation increased settlement of contested custody cases from 28% to 89%, halved time to reach agreement and increased satisfaction with outcome

...but does it work in theory?



- No clear role for uninformed mediator in dynamic bargaining
- *Complete info*: no role as already efficient
- *One sided private info*: approx same conclusion (Coase conjecture)
- *Two sided private info*: vast multiplicity of unmediated equilibria
  - B/c can "punish with beliefs" (identify deviator as weak type)
  - Range from very efficient (Ausubel & Deneckere ('93) achieve Myerson & Satterthwaite ('83) bounds) to almost no trade
  - Which equilibrium should mediation be compared to?

Main results

- Identify clear Pareto improvements from mediation in *reputational bargaining* model of Abreu & Gul ('00), two sided private info
  - Part 1 (*Mediation in reputational bargaining* (AER, 2021)): Using simple communication protocols close to those actually used by mediators
    - Importance of noise, small(ish) likelihood of commitment

Model

- AG but with pre-concession game communication/mediation stage
  - Agents choose initial demands
  - Rational agents can then "compromise": send private message to mediator
  - Mediator sends public message (suggesting agreement) with probability $b \in [0, 1]$ iff both agents privately compromise
    - $b = 0$ unmediated bargaining
    - $b = 1$ simple mediation
    - $b \in (0, 1)$ noisy mediation. Source of noise: agent messages go astray/misunderstood?
  - Agents can then change demands, before concession game
- Filter information: don't lose reputation if only you compromise
- Professional mediators claim to beneficially use such protocols

Initial observations/definitions

- After mediator message any continuation payoffs (m_1, m_2) possible
- ρ_i is prob rational i compromises
  - $P_j = (1 - z_j)b\rho_j$ is prob of mediator deal if i compromises
- $G_i^c(t)$ is prob rational i concedes by t if compromised+no deal
- $G_i^n(t)$ is prob rational i concedes by t if didn't compromise
  - $F_j^c(t)$ is prob j concedes by t if i compromised+no deal
  - $F_j^n(t) = P_j G_j^c(t) + (1 - P_j)F_j^c(t)$ is prob j concedes by t if i didn't compromise

First result

Proposition 6

Simple Mediation, $b = 1$, never works (same as unmediated outcome)

Why?

- Information is still released if no deal suggested
  - Agent who compromised becomes more pessimistic (opponent probably committed?)
  - Severe equilibrium effect: agent must immediately concede
  - Destroys opponent's incentive to compromise as $m_i < \alpha_i$

First result

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  - Agent who compromised becomes more pessimistic (opponent probably committed?)
  - Severe equilibrium effect: agent must immediately concede
  - Destroys opponent's incentive to compromise as $m_i < \alpha_i$
- Focus on symmetric parameters ($r_i = r$,$\alpha_i = \alpha$,$z_i = z$), equilibrium
  - Standard reasoning: continuous concession after time 0
  - Compromising agent knows she faces non-compromiser: if concedes on (s, s'), then so must non-compromiser

$$\frac{f^c(t)}{1 - F^c(t)} = \frac{f^n(t)}{1 - F^n(t)} = \lambda = \frac{r(1-\alpha)}{2\alpha - 1}$$

  - $F^n(t) = PG^c(t) + (1-P)F_j^c(t)$ then implies $\frac{g^c(t)}{1-G^c(t)} = \lambda$
  - And so $(1 - G^c(t)) \geq (1 - G^c(0))e^{-\lambda t} > 0$. But can't bargain forever (standard logic)

Noisy mediation

Proposition 7

When commitment is similarly small for both agents noisy mediation ($b \in (0,1)$) can strictly improve both rational agents' payoffs*

*For any $L \geq 1$, $\exists \bar{z} > 0$ s.t. if $z_i < \bar{z}$ and $z_i/z_j \in [1/L, L]$

Why?

- Again: focus on symmetric model, unmediated payoffs $(1-\alpha)$
- Assume all rational agents compromise
- As $b < 1$, need not concede after no deal (may just be unlucky)
  - Continuation play as in unmediated game but with updated reputations, $\bar{z} = \frac{z}{1-(1-z)b}$
- If z small, so is $\bar{z}$. Agent expects similar opponent concession rate whether she compromised or not. Continuation payoff$\approx (1-\alpha)$

$$U^c(T^*) - U^n(T^*) = P\left(m_i - \int_{s<t} \alpha e^{-rs} dG^c(s)\right) = \frac{P}{1-\bar{z}}\left((1-\bar{z})m_i - (1-\alpha)(1-\bar{z}^{\frac{\alpha}{1-\alpha}})\right)$$

- Compromise clearly beneficial when $\bar{z} \approx 0$
- Argument extends to asymmetric problems: adjust m_i to compensate stronger agent

Optimal dynamic mediation (2023)

- *Symmetric WOA:* 2 alternatives A (preferred by 1) and B (preferred by 2)
  - "Flexible" types get $\alpha \in (0,1)$ from preferred, $1-\alpha$ from other
  - "Commitment" types get α from preferred, $-\beta < 0$ from other
- Arbitrary communication: public/private messages any time
- Equilibrium outcomes described by:
  - $G(t)$ = prob two flexible types agree by t
  - p_i^t = prob flexible i concedes in such an agreement at t
  - $H_i(t)$ = prob flexible i concedes to committed j by t

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  - $G(t)$ = prob two flexible types agree by t
  - p_i^t = prob flexible i concedes in such an agreement at t
  - $H_i(t)$ = prob flexible i concedes to committed j by t
- All equilibria characterized by three incentive constraints (iff):
  - (Flexible) Obedience Constraint: Eq strategy weakly better than following it up to time t before conceding
  - Flexible Revelational Constraint: Eq strategy weakly better than acting as commitment type up to time t before conceding
  - Committed Revelational Constraint: Eq strategy weakly better than acting as flexible type but never conceding
- Wlog to restrict attention to *direct mediation protocols*:
  - Rational agents immediately reveal type to mediator who later publicly suggests agreement

Optimal mediation

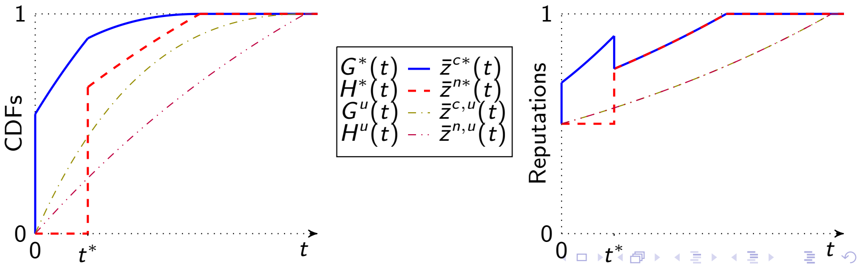
- *Mediator's problem:* maximize sum of flexible types' payoffs
- Immediately simplifies: can restrict attention to *symmetric* mediation protocols
  - Same distribution of agreement times/terms for each agent ($H_i = H$, $p_i^t = 1/2$)

Optimal mediation

Theorem 2

A unique Optimal (Symmetric) Mediation Protocol (G, H) exists

- *Distribution of agreement times b/w two flexible agents, G:*
  - *Has atom at time 0, then increases continuously to make flexible agents indifferent to conceding on $(0, T]$*
- *Distribution of agreement times b/w flexible and committed, H:*
  - *Has atom at time $t^* \geq 0$, then increases continuously to make deviating flexible agent indifferent to conceding on $(t^*, T]$*
- *Improves unmediated bargaining iff $z < \alpha$ (then reputation α at t^*)*



When is mediation beneficial?

Mediation can improve on unmediated bargaining iff $z < \alpha$

- From a flexible-committed agreement, flexible agent gets:
  - $(1 - \alpha)$ with probability z if honest (report flexible)
  - α with probability $(1 - z)$ if dishonest
- If $(1 - \alpha)z \geq \alpha(1 - z)$, i.e. $z \geq \alpha$, then delaying these agreements is more costly when honest$\Rightarrow$will pretend to be committed
- Logic also explains why reputations must equal α at t^*
  - Further delay of flexible-committed agreements more costly to flexible agent when honest

Arbitration: mechanism design benchmark

- Allow designer to impose (immediate) agreement and/or (perpetual) disagreement based on type reports
  - Revelation constraint: truthfully reveal type to designer
  - Interim participation constraint: payoff larger than unmediated bargaining

Arbitration: mechanism design benchmark

Proposition 8

Unique optimal symmetric arbitration exists and provides higher flexible payoffs than optimal mediation:

(1) *Selects each alternative with prob 1/2 if that satisfies committed participation constraint (e.g.* $\alpha \geq (\beta + 2)/3$*)*

(2) *If not:*

- *Flexible type pairs always agree*
- *If* $z \geq 2\alpha - 1$*, flexible types always agree with commitment types and get higher payoffs than in complete info bargaining*
- *Flexible type sometimes gets preferred alternative with commitment type*
- *Commitment type pairs sometimes disagree (always if* $\alpha \leq \beta$*)*

Arbitration: mechanism design benchmark

Proposition 8

Unique optimal symmetric arbitration exists and provides higher flexible payoffs than optimal mediation:

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- *Flexible type sometimes gets preferred alternative with commitment type*
- *Commitment type pairs sometimes disagree (always if $\alpha \leq \beta$)*

- **Problem?** if (1) impossible and can't impose *disagreement* (who has such standing?) then:
  - Flexible types would initially pretend to be committed before later conceding⇒back to mediation

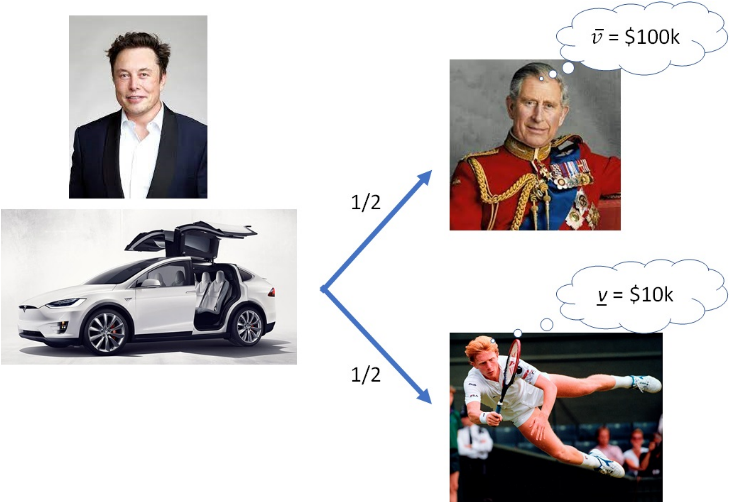
Story of mediation paper

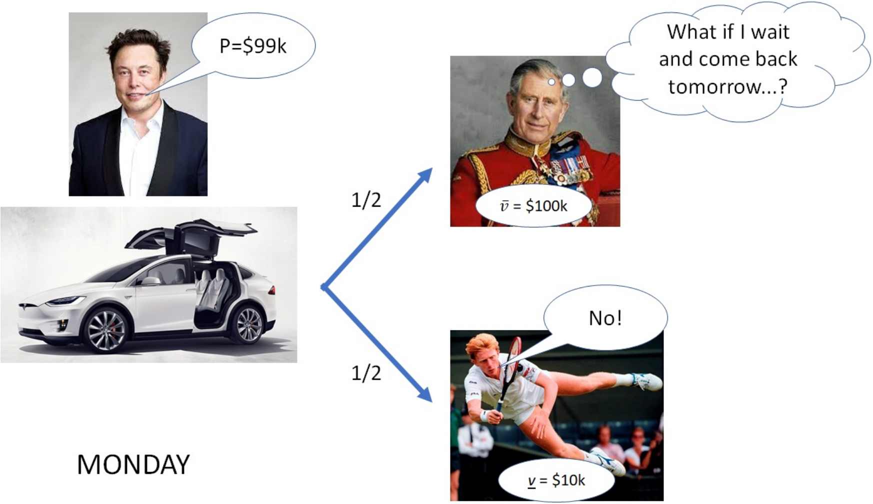
- Had idea of investigating mediation at very end of grad school
- Year 1 at Brown: first negative result (ineffectiveness of simple mediation)
- Year 2 at Brown: second positive result (noisy mediation effective)
  - Pushed to solve optimal mediation case: Larry Samuelson, Joel Watson
- Year 3/4 at Brown: optimal mediation solvable with symmetry!
- Broken into two papers at request of AER editor (Jeff Ely)
  - Rational commitment types in WOA at suggestion of referees

Outside options, reputations, and the partial success of the Coase conjecture (R&R ECMA)

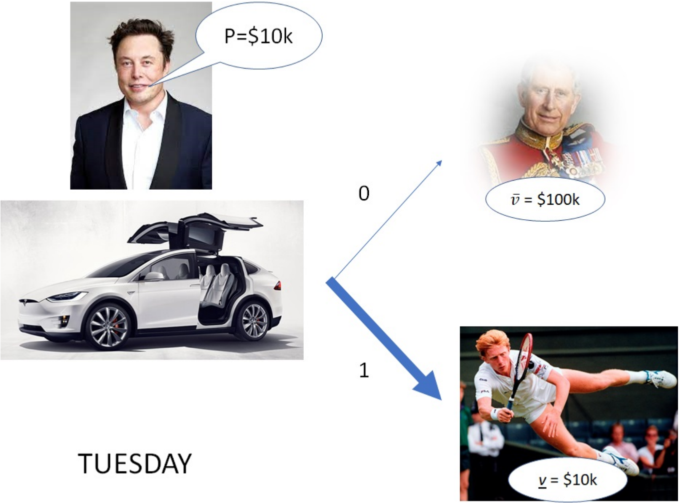
Origin story: What is effect of private buyer value in bargaining?

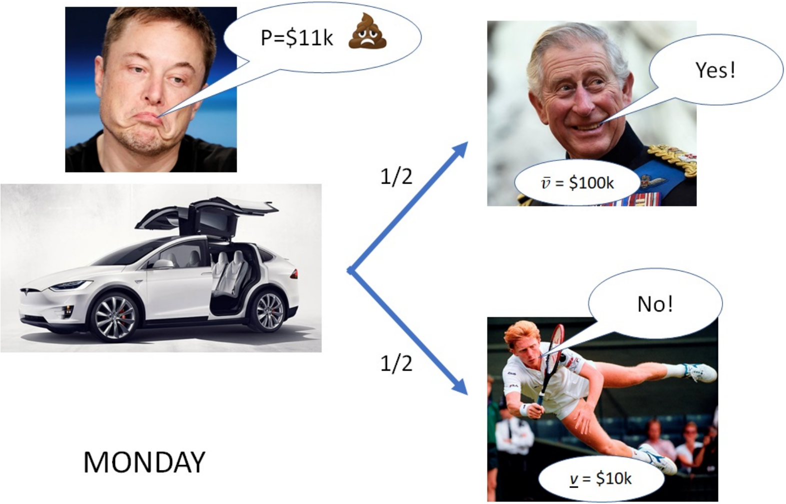
- Seller makes repeated offers to buyer w/ private value $v \in [\underline{v}, \overline{v}]$
  - $\Rightarrow$ *Coase conjecture:* Seller charges $\underline{v}$ almost immediately if frequent offers
  - Due to negative selection: Buyers who don't accept today have low values, so seller reduces price tomorrow
    - *Competes w/ future self:* high value buyers won't accept today either unless low price
  - *Gul et al ('86), Fudenberg et al ('85)*


$\bar{v}$ = $100k
1/2
1/2
$\underline{v}$ = $10k

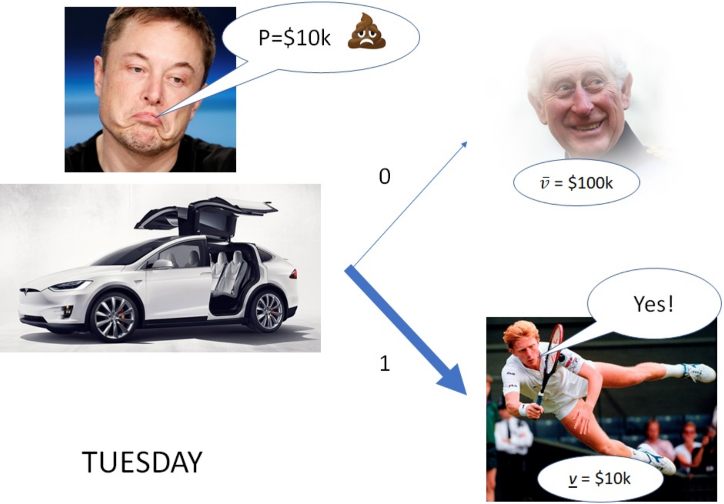

P=$99k
1/2
1/2
What if I wait and come back tomorrow...?
$\bar{v}$ = $100k
No!
$\underline{v}$ = $10k

MONDAY


P=$10k
0
$\bar{v}$ = $100k
1
$\underline{v}$ = $10k
TUESDAY



MONDAY


P=$10k
0
$\bar{v}$ = $100k
Yes!
1
$\underline{v}$ = $10k
TUESDAY

But what if buyer has an outside option?

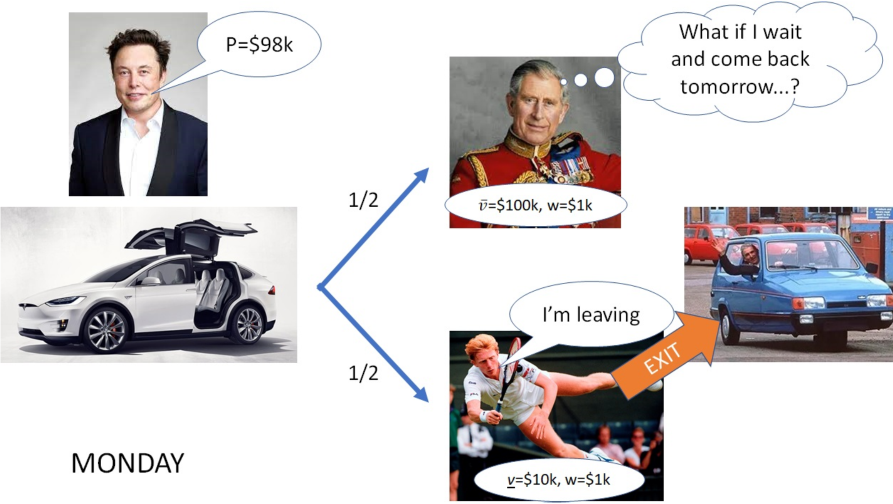
- Seller makes repeated offers to buyer w/ private value $v \in [\underline{v}, \overline{v}]$ and + outside option $w \in [\underline{w}, \overline{w}]$
  - $\Rightarrow$ Seller can commit to any take-it-or-leave-it offer
  - Due to positive selection: low value buyers exit today, so remaining buyers have high value
  - No surplus for type w/ lowest net value, $v - w$, that continues to period 2: cont. payoff$\leq w$
  - $\Rightarrow$ Better to exit in period 1
  - *Board & Pycia ('14)*

1/2

$\bar{v}$=\$100k, w=\$1k

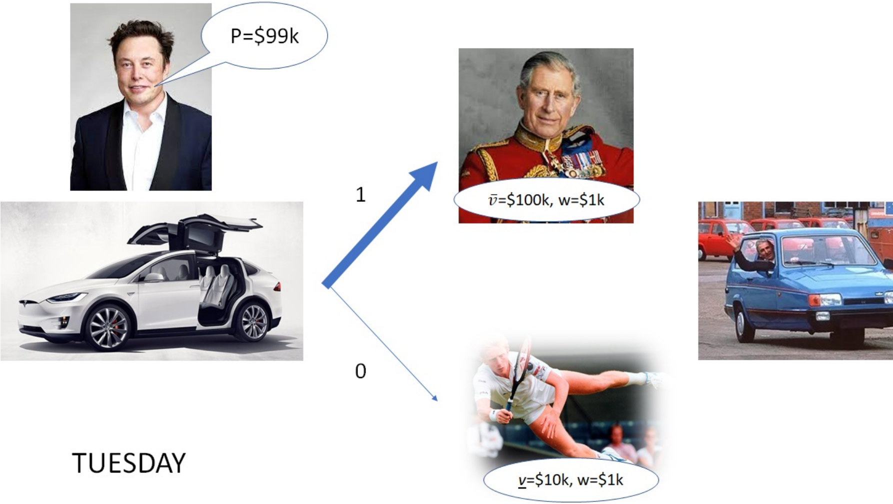
$\underline{v}$=\$10k, w=\$1k

1/2

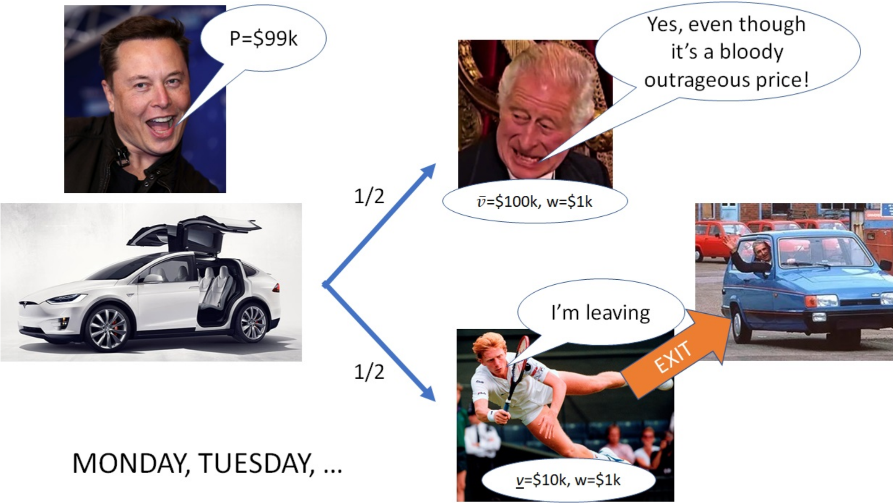


$P=\$98k$
1/2
1/2
What if I wait and come back tomorrow...?
$\bar{v}=\$100k, w=\$1k$
I'm leaving
EXIT
$\underline{v}=\$10k, w=\$1k$

MONDAY

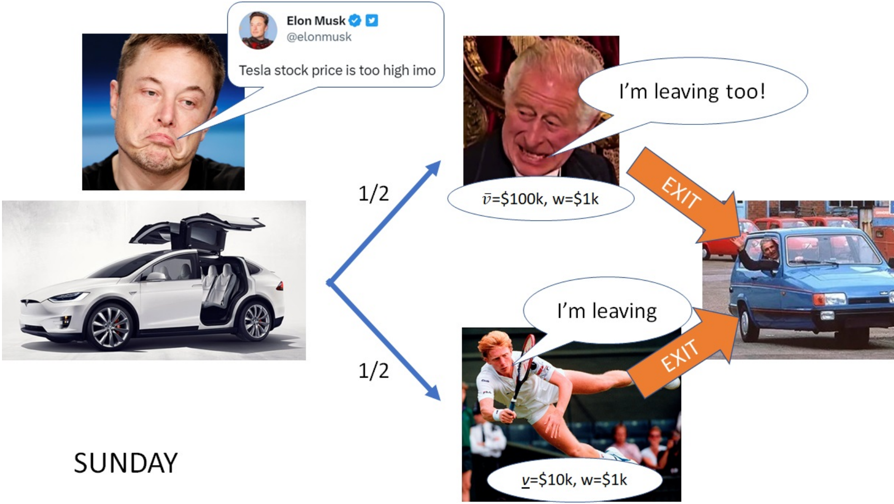


TUESDAY


P=$99k
Yes, even though it's a bloody outrageous price!
$\bar{v}$=$100k, w=$1k
1/2
1/2
I'm leaving
EXIT
$\underline{v}$=$10k, w=$1k
MONDAY, TUESDAY, ...

How robust is this prediction?

- Suppose buyer can now take + outside option $w \in [\underline{w}, \overline{w}]$ before bargaining (period 0)
  - $\Rightarrow$ Market unravels. No trade!
  - No surplus for type w/ lowest net value, $v - w$, that continues to period 1: cont. payoff$\leq w$
  - $\Rightarrow$ Better to exit in period 0


Elon Musk
@elonmusk
Tesla stock price is too high imo
1/2
1/2
I'm leaving too!
$\bar{v}$=$100k, w=$1k
EXIT
I'm leaving
EXIT
$\underline{v}$=$10k, w=$1k

SUNDAY

Avoiding the paradox?

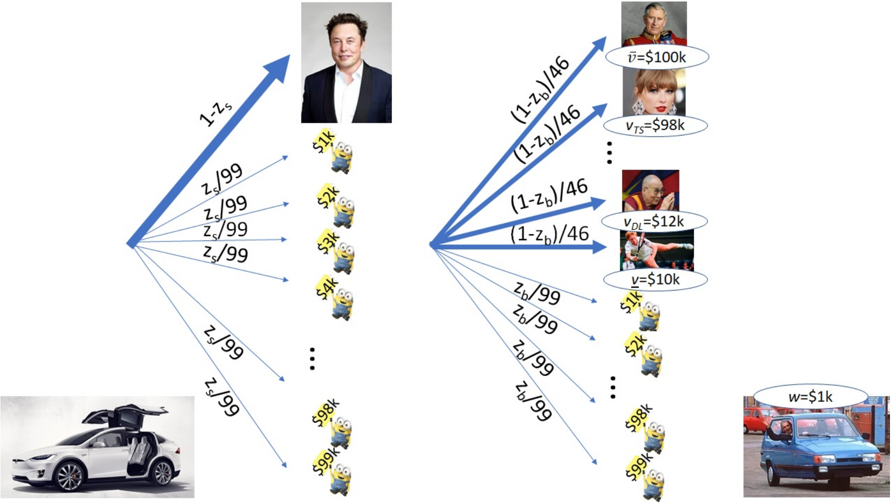
- To avoid unravelling paradox: allow some buyer offers? $\Rightarrow$ Surplus
  - But then signal private info: punish w/ beliefs...
- But what if each bargainer could also be commitment type?

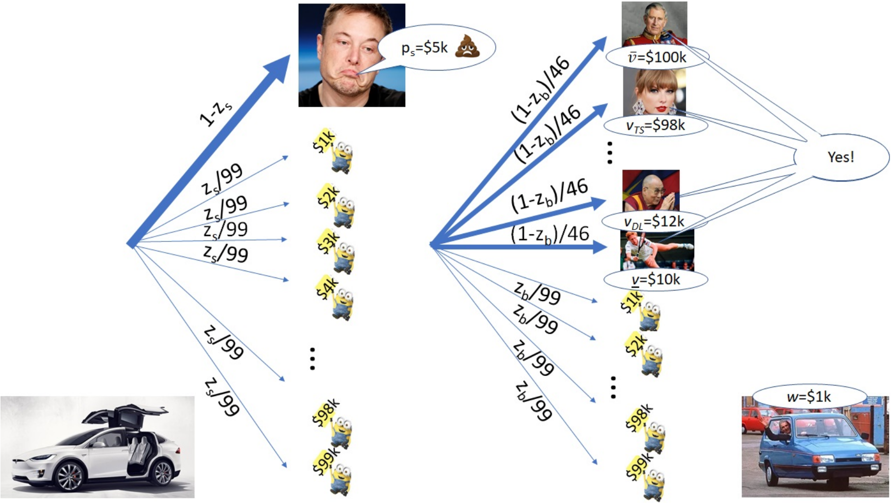
Main result: rich buyer values

Main result: if sets of buyer values and commitment types are rich and commitment vanishes then outcomes equivalent to seller choosing any ultimatum below $p^* = \max\{\underline{v}/2, \underline{w}\}$

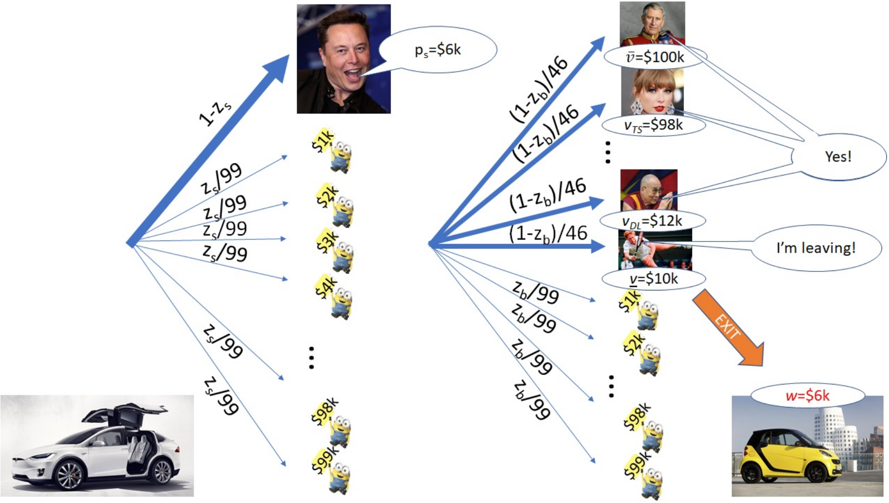
Partly Coasean:

- No delay + low prices if $\underline{v} \approx 0 \approx \underline{w}$
- But some seller market power and inefficiency as +ve net value buyers exit $p^* >> \min\{v - w > 0\}$
  - High prices if $\underline{v} >> 0$ or $\underline{w} >> 0$


1-z_s
z_s/99
z_s/99
z_s/99
z_s/99
z_s/99
z_s/99
$1k
$2k
$3k
$4k
$98k
$99k
(1-z_b)/46
(1-z_b)/46
(1-z_b)/46
(1-z_b)/46
z_b/99
z_b/99
z_b/99
z_b/99
v̄=$100k
v_TS=$98k
v_DL=$12k
v=$10k
$1k
$2k
$98k
$99k
w=$1k



p_s=\$5k
$1-z_s$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
\$1k
\$2k
\$3k
\$4k
\$98k
\$99k
$(1-z_b)/46$
$(1-z_b)/46$
$(1-z_b)/46$
$(1-z_b)/46$
$\bar{v}$=\$100k
v_{TS}=\$98k
v_{DL}=\$12k
$\underline{v}$=\$10k
Yes!
$z_b/99$
$z_b/99$
$z_b/99$
$z_b/99$
\$1k
\$2k
\$98k
\$99k
w=\$1k



p_s=$6k
$1-z_s$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
$z_s/99$
$1k
$2k
$3k
$4k
$98k
$99k
$(1-z_b)/46$
$(1-z_b)/46$
$(1-z_b)/46$
$(1-z_b)/46$
$\bar{v}$=$100k
v_{TS}=$98k
v_{DL}=$12k
$\underline{v}$=$10k
Yes!
I'm leaving!
$z_b/99$
$z_b/99$
$z_b/99$
$z_b/99$
$1k
$2k
$98k
$99k
EXIT
w=$6k

Logic for main result

Main result: if sets of buyer values and commitment types are rich and commitment vanishes then outcomes equivalent to seller choosing any ultimatum below $p^* = \max\{\underline{v}/2, \underline{w}\}$

- **Intuition:** Positive *and* negative selection:
    - Rational buyer who finds seller price unacceptable $w > v - p_s$ immediately exits
    - Lowest value remaining buyer, $v^1 = \min\{v > \underline{w} + p_s\}$, can get half of surplus (as equal bargaining power)
        - If $p_s > p^*$ then also $p_s > v^1/2$ and if $p_b \approx p^*$ then eventually $\lambda_b > \lambda_s$ so seller immediately concedes
    - $p^* =$ max price s.t. $p_s \leq v/2$ for all $v > \underline{w} + p_s$

More model

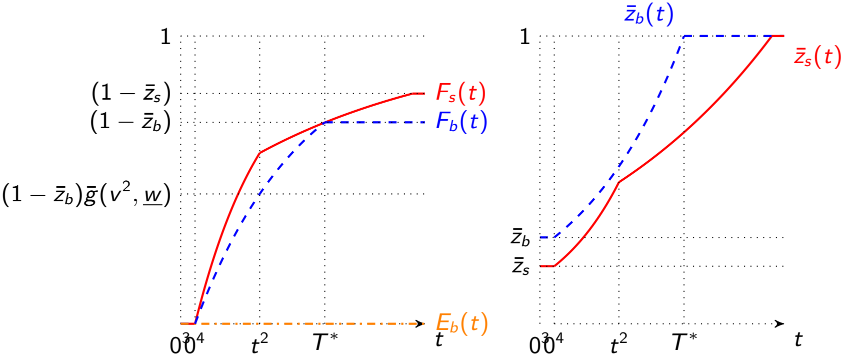
- Finite rational types (v, w). Prob> 0 of $(v, \underline{w})$ for each v
- Divide time 0 into times $0^1 < 0^2 < 0^3 < 0^4$ w/o discounting b/w
  - At 0^1, seller proposes a price p_s from finite set P
  - At 0^2, buyer can accept (concede), counterdemand $p_b < p_s$ from P or exit game
  - If game continues to 0^3 agents choose stopping time to concede, or exit (potentially for the buyer) $t_i \in \{0^3, 0^4\} \cup (0, \infty]$
- $F_i^{p_s,p_b}(t) =$ prob agent i concedes by time t
- $E_b^{p_s,p_b}(t) =$ prob buyer exits by time t

WOA: 2 rational buyer types

- Suppose commitment demands $p_s \in P_s$, $p_b \in P_b$
- Rational buyers $(v^1, \underline{w})$, $(v^2, \underline{w})$ with $v^2 > v^1$, $v^i - p_s > \underline{w}$
- Unique equilibrium characterized by:
  1. At most one agent concedes with positive probability at time 0
  2. Agents reach a probability 1 reputation at same time $T^* < \infty$
  3. Skimming property: buyer v^2 concedes before buyer $v^1 < v^2$
     - Agents concede at rates $(\lambda_s^{v^2}, \lambda_b)$ on $(0, t^2)$, and $(\lambda_s^{v^1}, \lambda_b)$ on (t^2, T^*) so opponent indifferent b/w conceding and waiting

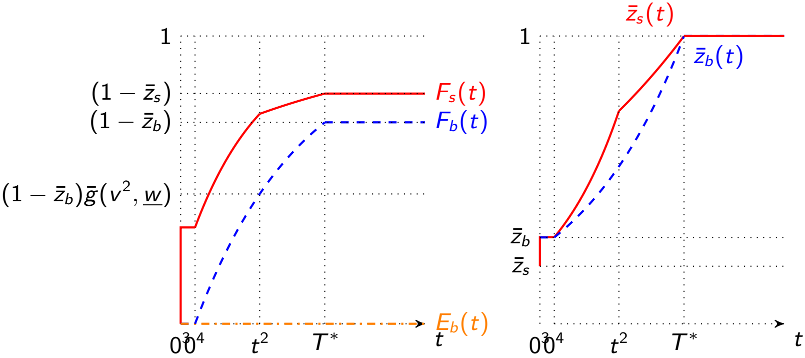
$$\lambda_s^v := \frac{r(v - p_s)}{p_s - p_b}, \qquad \lambda_b := \frac{rp_b}{p_s - p_b}.$$

Reputational race: if no time 0 concession



$$\lambda_s^v := \frac{r(v-p_s)}{p_s-p_b}, \qquad \lambda_b := \frac{rp_b}{p_s-p_b}.$$

Reputational race: adjust time 0 concession



$$\lambda_s^v := \frac{r(v - p_s)}{p_s - p_b}, \qquad \lambda_b := \frac{rp_b}{p_s - p_b}.$$

WOA: 3 rational buyer types

- Add third rational buyer type (v, w) which prefers to exit $w > v - p_s$:
  - Indifferent b/w exit and waiting if seller concedes at rate:

$$\underline{\lambda}^{v,w} := \frac{rw}{v - p_b - w}$$

WOA: 3 rational buyer types

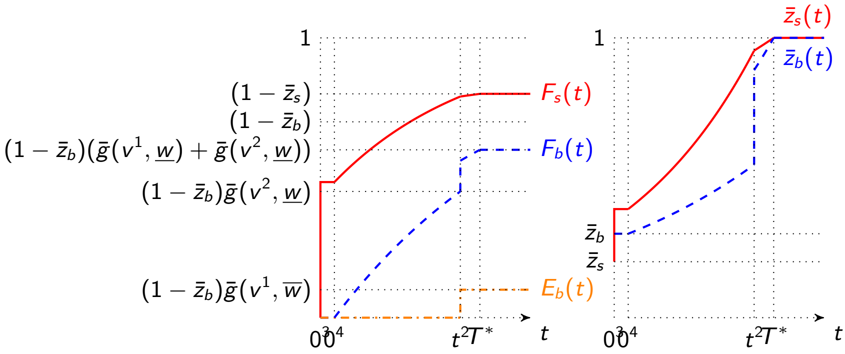
- Add third rational buyer type (v, w) which prefers to exit $w > v - p_s$:
  - Indifferent b/w exit and waiting if seller concedes at rate:

$$\underline{\lambda}^{v,w} := \frac{rw}{v - p_b - w}$$

- Any equilibrium in concession game characterized by:
  2. Agents reach a probability 1 reputation at same time $T^* < \infty$
  3. Skimming property: buyer v^2 concedes before buyer $v^1 < v^2$
     - Agents concede at rates $(\lambda_s^{v^2}, \lambda_b)$ on $(0, t^2)$, and $(\lambda_s^{v^1}, \lambda_b)$ on (t^2, T^*) so opponent indifferent b/w conceding/waiting
  4. Buyer (v, w) exits at:*
     - 0^4 if $\underline{\lambda}^{v,w} > \lambda_s^{v^2}$,
     - t^2 if $\underline{\lambda}^{v,w} \in (\lambda_s^{v^1}, \lambda_s^{v^2})$
     - T^* if $\underline{\lambda}^{v,w} < \lambda_s^{v^1}$

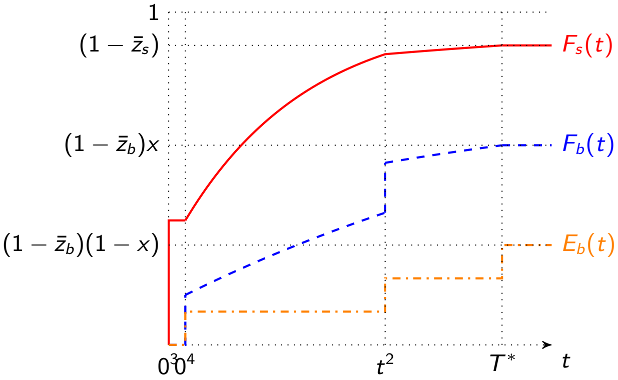
* If buyer exits at $t < T^*$, must also concede at t to ensure seller waits until T^*

Equilibrium with some exit



- Here: $\lambda_s^{v^2} > \underline{\lambda}^{v,w} > \lambda_s^{v^1}$

Add more types



- Where x is prob. rational buyer eventually concedes: $v - p_s > w$
- Seller concession at 0^3 is consistent w/ buyer exit+concession at 0^4
- Equilibrium need not be unique

Demand choice: buyer at 0^2

Preference for low prices

(i) Buyers who concede, $v - p_s > w$, weakly prefer low price $p_b' \in P_b$ to $p_b > p_b'$

(ii) Buyers who exit, $w > v - p_s$, only ever demand $\underline{p} = \min P \in P_b$

Demand choice: buyer at 0[2]

Preference for low prices

(i) Buyers who concede, $v - p_s > w$, weakly prefer low price $p_b' \in P_b$ to $p_b > p_b'$

(ii) Buyers who exit, $w > v - p_s$, only ever demand $\underline{p} = \min P \in P_b$

Why?

i) High value buyers indifferent b/w subset of demands. Implies slower seller concession after low demands, such delay less costly for lower values

ii) Gain from exit vs concession, $[w - (v - p_s)]e^{-rt}(1 - F_s(t)) > 0$, larger if less seller concession (as occurs after low demands)

WOA: 2 rational buyer types, commitment vanishes

$v^2 > v^1$ *and* $v^1 - p_s > \underline{w}$

- Seller immediately concedes ($F_s(0^4) \to 1$) if less generous than v^1 buyer $v^1 - p_s < p_b$
- Buyer immediately concedes ($F_b(0^4) \to 1$) if v^1 type less generous than seller $p_b < v^1 - p_s$

WOA: 2 rational buyer types, commitment vanishes

$v^2 > v^1$ *and* $v^1 - p_s > \underline{w}$

- Seller immediately concedes ($F_s(0^4) \to 1$) if less generous than v^1 buyer $v^1 - p_s < p_b$
- Buyer immediately concedes ($F_b(0^4) \to 1$) if v^1 type less generous than seller $p_b < v^1 - p_s$

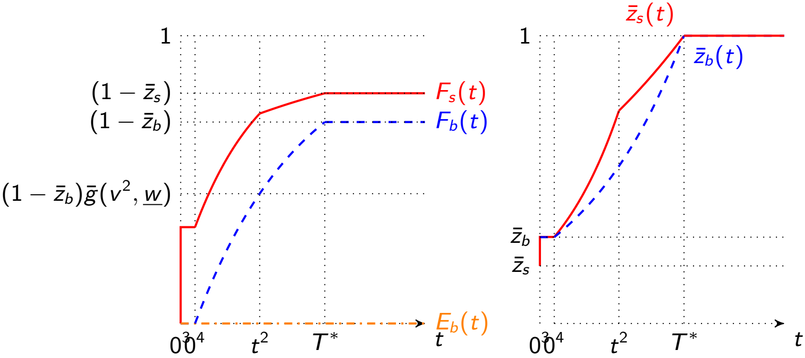
Only lowest value v^1 matters. Coasean logic. Why?

- As in Abreu&Pearce, my deadline paper: only LR delay costs matter
- Concession exhausts higher value buyers quickly ($\lim t^2 < \infty$), then $\lambda_b > \lambda_s^{v^1}$ if buyer more generous

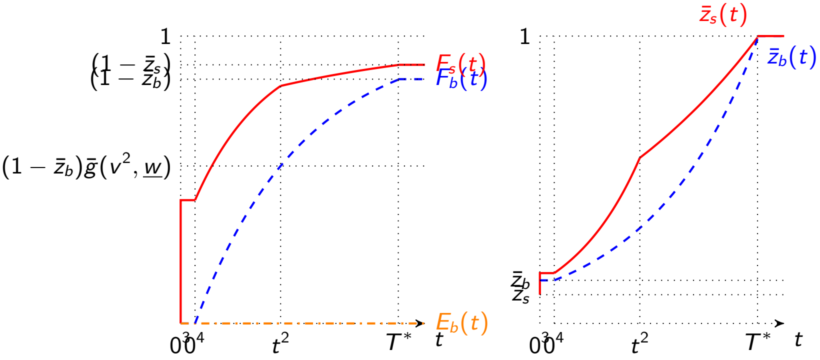
$$\lambda_s^v := \frac{r(v - p_s)}{p_s - p_b}, \qquad \lambda_b := \frac{rp_b}{p_s - p_b}.$$

- Reputations still small after high value buyers exhausted $\bar{z}_i(t^2) \approx 0$
- Exponentially faster reputational growth afterwards, means less generous agent must concede immediately w large prob so both reach reputation=1 at T^*

Recall: 2 rational buyer types



Halve initial reputations: 2 rational buyer types



- Seller must concede at 0 more often, $F_s(0^4) = 0.43 > 0.33$, to reach prob.1 reputation at T^*
- *Increased Coasean force:* Increasingly only v^1 matters: spend greater share of time after t^2 ($t^2/T^* \to 0$) where buyer is more generous than seller $v^1 - p_s > p_b$ and so builds reputation faster

WOA: 3 rational buyer types as commitment vanishes

Additional type with $w > v - p_s$*: can only demand* $\underline{p}$

- Must have $\underline{p} \approx 0$ when rich set of commitment types
- Extremely ungenerous: seller will wait to concede if even small prob. of subsequent buyer concession (given $p_s > 0$, $v - p_s > \underline{w} > 0$)
  - $E_b(0^4) + F_b(0^4) \to 1$, *buyer immediately concedes or exits*

Almost there: main result

Proposition 9 (Informal: seller payoffs)

If the distribution of agents' types are rich enough, and probability of commitment small enough, then outcomes approximate those where seller makes ultimatum with upper bound on prices of $p^* = \max\{\underline{v}/2, \underline{w}\}$.

Almost there: main result

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- *Definition:* a rational buyer's distribution of types (g, Θ) is $\varepsilon > 0$ rich if for any $a \in [\underline{v}, \overline{v}]$, $|v - a| < \varepsilon$ for some $v \in V$.
- *Definition:* Given a rational buyer's type distribution, agents' commitment type distributions are $\varepsilon' > 0$ rich if for any $a \in [0, \overline{v} - \underline{w}]$, for each i, $|p_i - a| < \varepsilon'$ for some $p_i \in P_i$
- Equivalence when $z_i \to 0$ then $\varepsilon' \to 0$ then $\varepsilon \to 0$

What's special about p^*? An inflection point of generosity

- $p^* = \max\{\underline{v}/2, \underline{w}\}$ is largest demand where seller can guarantee she's *more generous* than lowest buyer who eventually concedes, $v^{1,p_s} = \min\{v \geq \underline{w} + p_s\}$
  - If $p_s \leq \underline{v}/2$ then $p_b < p_s \leq \underline{v}/2 \leq \underline{v} - p_s \leq v^{1,p_s} - p_s$
  - If $p_s \leq \underline{w}$ then $p_b < p_s \leq \underline{w} < v^{1,p_s} - p_s$
  - If $p_s > p^*$ then v^{1,p_s} could counterdemand $p_b \approx p^*$ where $p_b > v^{1,p_s} - p_s$
- Being more generous than lowest value buyer who concedes is *ALL* that matters as commitment vanishes!

Extensions

- Rich set of buyer values needed for result
  - With binary values $v \in \{\underline{v}, \overline{v}\}$ seller can potentially charge $p_s \approx \overline{v}/2 >> p^*$
  - *Positive selection:* Low value buyer $\underline{v} < \underline{w} + p_s$ immediately exits
- Extend results to discrete time alternating offers game
- Extend to different discount rates: if $r_b >> r_s$ then seller can make any ultimatum!
- Seller can benefit from larger buyer outside option/sunk costs/initial delay:
  - Increases positive selection


THE END